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Lepton Mass Hierarchy and Neutrino Oscillations

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Abstract

Starting from the symmetry of lepton flavor democracy, we propose and discuss a simple pattern for the mass generation and flavor mixing of the charged leptons and neutrinos. The three neutrino masses are nearly degenerate, and the flavor mixing angles can be calculated. The observed deficit of solar and atmospheric neutrinos can be interpreted as a consequence of the near degeneracy and large oscillations of ν_e, ν_μ and ν_τ in the vacuum. Our *Ansatz* can also accommodate the cosmological requirement for hot dark matter and the current data on neutrinoless $\beta\beta$-decay.

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In the standard model of the electroweak interactions the masses of quarks and leptons as well as the flavor mixing angles enter as free parameters. Further insights into the yet unknown dynamics of the mass generation require steps beyond the standard model. The first one in this direction could be the identification of specific patterns and symmetries and the associated symmetry breaking.

Recently a number of authors [1, 2, 3] have stressed that the observed hierarchies in the lepton-quark mass spectrum could be interpreted as a hint towards the significance of a "democratic" mass matrix both for the up- and down-type quarks and for the charged leptons:

$$M_{0i} \;=\; c_i \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{1}$$

(i stands for u, d in case of quarks and l in case of the charged leptons). These mass matrices are supposed to be valid in the limit where the first and second family of leptons and quarks are massless. Small violations of the "democratic symmetry" can account for the masses of the second and first family of quarks as well as for the flavor mixing angles (see, e.g. refs. [2, 3]).

In this paper we should like to point out that an application of similar ideas to the leptons can lead to a surprisingly simple pattern for the mass generation and flavor mixing in the neutrino sector. In particular the mixing angles describing neutrino oscillations are large, calculable and consistent with experimental constraints.

In the "democratic limit" only the third family of quarks and leptons (i.e. the (t, b) system and the τ-lepton) acquire masses. Suppose a mass would also be introduced for the τ-neutrino along the same line. In this case we would obtain a massive neutrino ν_τ, which could be either a Majorana or a Fermi-Dirac state, and the neutrino mass matrix takes the form:

$$M_{0\nu} \;=\; c_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \;, \tag{2}$$

($m_{\nu_\tau} = 3c_\nu$). Since according to astrophysical constraints the ν_τ-state must be very light, i.e. not heavier than about 30 eV, we would have a situation in which the constants c_ν and c_l for the various flavor channels differ by at least eight orders of magnitude ($c_\nu/c_l < 30$ eV$/m_\tau \sim 10^{-8}$). We find that such a tiny ratio is very unnatural, and one is invited to look for another possibility to introduce the neutrino masses.

In our view the simplest way to avoid the problem mentioned above is to suppose that the constant c_ν vanishes, i.e. the neutrinos do not receive any mass contribution in the "democratic

limit". We do not attempt to discuss the dynamical reason for the vanishing of c_ν except for mentioning that it would follow if one could establish a multiplicative relation between the fermion masses in the "democratic limit" and their electric charges, i.e. the vanishing of c_ν would be directly related to the fact that the neutrinos are electrically neutral. If c_ν vanishes, it is automatically implied that there exists a qualitative difference between the neutrino sector and the charged lepton sector. In particular it is expected that the neutrino masses are small compared to the main entry in the charged lepton mass matrix $c_l = m_\tau/3$, and in particular there would be no reason why the hierarchical pattern observed for the charged lepton masses should repeat itself for the neutrino masses, i.e. the three neutrino masses could be of the same order in magnitude.

In the absence of the "democratic" neutrino term, one would have:

$$M_l \; = \; c_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_l \; , \qquad M_\nu \; = \; 0 + \Delta M_\nu \; , \tag{3}$$

where ΔM_l and ΔM_ν are the symmetry breaking terms for the charged leptons and neutrinos, respectively. As discussed previously [4], a simple breaking term would be a diagonal mass shift for the "democratic" eigenstates, i.e.

$$\Delta M_l \; = \; \begin{pmatrix} \delta_l & 0 & 0 \\ 0 & \varrho_l & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix} \; , \qquad M_\nu \; = \; \begin{pmatrix} \delta_\nu & 0 & 0 \\ 0 & \varrho_\nu & 0 \\ 0 & 0 & \varepsilon_\nu \end{pmatrix} \; . \tag{4}$$

Both M_l and M_ν are real matrices, i.e. CP symmetry is preserved for the leptons. The neutrino mass matrix is already diagonal (eigenvalues: $\delta_\nu, \varrho_\nu, \varepsilon_\nu$), while the mass matrix for the charged leptons needs to be diagonalized. Apart from small corrections from ΔM_l, the main effect of the diagonalization is to diagonalize the "democratic matrix" M_{0l} by the transformation $UM_{0l}U^\dagger = M^l_{\rm H}$, where $M^l_{\rm H}$ is the "hierarchical" matrix:

$$M^l_{\rm H} \; = \; c_l \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \; , \qquad U \; = \; \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \; . \tag{5}$$

In a good approximation the leptonic flavor mixing matrix is given by the matrix U above, i.e. the leptonic doublets are given by

$$\begin{pmatrix} \frac{1}{\sqrt{2}}(\nu_1 - \nu_2) & \frac{1}{\sqrt{6}}(\nu_1 + \nu_2 - 2\nu_3) & \frac{1}{\sqrt{3}}(\nu_1 + \nu_2 + \nu_3) \\ e^- & \mu^- & \tau^- \end{pmatrix} \; , \tag{6}$$

where ν_1, ν_2, ν_3 are the neutrino mass eigenstates.

In order to estimate the corrections from the symmetry breaking term ΔM_l, we consider an illustrative example, in which $\delta_l = -\varrho_l$ is taken. This case is of particular interest. The mass matrix in the hierarchical basis, i.e. the matrix $M_{\rm H}^l$, has a vanishing (1,1) element, and the mixing angles can be completely expressed in terms of mass eigenvalues. Taking into account the higher order terms from the symmetry breaking, one finds for the lepton mixing matrix V:

$$V \; = \; U + \sqrt{\frac{m_e}{m_\mu}} \left(\begin{array}{ccc} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{array} \right) + \frac{m_\mu}{m_\tau} \left(\begin{array}{ccc} 0 & 0 & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{array} \right) . \tag{7}$$

Here $\sqrt{m_e/m_\mu} \approx 0.0696$ and $m_\mu/m_\tau \approx 0.0594$ [5]. In general the real mixing matrix V can be parametrized in terms of three Euler angles, which we define in analogy to the quark mixing angles [5]:

$$V \; = \; \left(\begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{array} \right) \tag{8}$$

($c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$). Comparing eqs. (7) and (8), the three mixing angles can be determined as follows:

$$\tan\theta_{12} = -1 + \frac{2}{\sqrt{3}}\sqrt{\frac{m_e}{m_\mu}} \; , \qquad \tan\theta_{23} = -\sqrt{2} - \frac{3}{\sqrt{2}}\frac{m_\mu}{m_\tau} \; , \qquad \tan\theta_{13} = -\frac{2}{\sqrt{6}}\sqrt{\frac{m_e}{m_\mu}} \; . \tag{9}$$

The angle θ_{13} is very small, compared to θ_{12} and θ_{23}, and vanishes in the limit $m_e \rightarrow 0$. In this special case one obtains a two-angle parametrization of V.

In the more general case, where $\delta_l \neq -\varrho_l$, the mixing angles cannot be calculated only in terms of the mass eigenvalues. Nevertheless, the pattern that V is very close to U persists. In the following we shall use $V = U$ to discuss neutrino oscillations and the associated problems. We emphasize that in this limit the mixing angles are algebraic numbers independent of the lepton masses. The three angles can all be arranged to be in the second quadrant: $\theta_{12} = 135^0$, $\theta_{23} = 125.3^0$ and $\theta_{13} = 180^0$. In particular the electron neutrino ν_e is a mixture of the two mass eigenstates ν_1 and ν_2, with maximal mixing between the two. Both ν_μ and ν_τ are composed of all three mass eigenstates (see eq. (6)). It is useful to compare the structure of the neutrino states with the $SU(3)$ wave functions of the pseudoscalar mesons in the chiral limit of $SU(3)_L \times SU(3)_R$ symmetry [4]. In terms of $(\bar{q}q)$ states (q: quark field), one has:

$$\pi^0 \; = \; \frac{1}{\sqrt{2}}|\bar{u}u - \bar{d}d\rangle \; , \qquad \eta \; = \; \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle \; , \qquad \eta^{'} \; = \; \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle \; . \tag{10}$$

Thus the flavor eigenstates ν_e, ν_μ and ν_τ correspond to π^0, η and η', and the neutrino mixing angles are identical to the mixing angles of the pseudoscalar mesons in the chiral limit.

Analogous to the quark fields, the lepton fields enter the weak charged current in a mixed form described by V [6]. Neutrino beams will oscillate. Assuming a ν_i to have been produced at proper time $t = 0$, with momentum $\mathbf{P}$ ($|\mathbf{P}| >> m_i$, $i = 1, 2, 3$), then the probability for observing a ν_l generating from ν_i at time t is given by

$$P_{li} = \sum_{k=1}^{3} (V_{ik}V_{lk})^2 + 2\sum_{k>j} \{(V_{ij}V_{lj}V_{ik}V_{lk}) \cos [(E_k - E_j)\, t]\} \ . \tag{11}$$

In the approximation of $V = U$, we obtain:

$$P_{\mu e} = \frac{1}{6} - \frac{1}{6}\cos [(E_2 - E_1)t] \ , \qquad P_{\tau e} = \frac{1}{3} - \frac{1}{3}\cos [(E_2 - E_1)t] \ , \tag{12a}$$

and

$$P_{\tau\mu} = \frac{1}{3} + \frac{1}{9}\cos [(E_2 - E_1)t] - \frac{2}{9}\cos [(E_3 - E_1)t] - \frac{2}{9}\cos [(E_3 - E_2)t] \ . \tag{12b}$$

By use of $E_i \approx |\mathbf{P}| + m_i^2/(2|\mathbf{P}|)$, the above transition probabilities can be expressed in terms of the quantities $F_{ij} = 1.27\Delta m_{ij}^2 L/|\mathbf{P}|$, where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ (in unit of eV^2) denotes the mass-squared difference of neutrinos and L (in unit of km/GeV or m/MeV) is the distance from the neutrino's production point to its interaction point. One obtains:

$$P_{\mu e} = \frac{1}{3}\sin^2 F_{21} \ , \qquad P_{\tau e} = \frac{2}{3}\sin^2 F_{21} \ , \tag{13a}$$

and

$$P_{\tau\mu} = -\frac{2}{9}\sin^2 F_{21} + \frac{4}{9}\sin^2 F_{31} + \frac{4}{9}\sin^2 F_{32} \ . \tag{13b}$$

Note that $P_{li} = P_{il}$ is a consequence of CP symmetry, and the unitarity of U implies $P_{ei} + P_{\mu i} + P_{\tau i} = 1$ for each ν_i ($i = e$, μ or τ).

Let us confront the above results with the solar and atmospheric neutrino experiments. The experimental data of neutrino oscillations are usually presented as allowed regions on a $\Delta m_{ij}^2 - \sin^2\Theta_{ij}$ plot in the assumption of two-flavor oscillations, although there are (at least) three flavors of neutrinos. Here the mixing angles Θ_{ij} ($i, j = 1, 2, 3$), which can be derived from θ_{ij} in the lepton mixing matrix (8), do measure the magnitudes of neutrino oscillations. The current evidence for the deficit of solar neutrinos mainly comes from four experiments [7]. Such a deficiency can be interpreted as a ν_e-neutrino disappearance experiment due to its oscillations to ν_μ and ν_τ neutrinos. From eq. (13) we obtain

$$P_{ee} = 1 - \sin^2 F_{21} \ , \tag{14}$$

which incorporates the maximal mixing $\sin^2 2\Theta_{21} = 1$. Analyses of solar neutrino data on the basis of two-neutrino oscillations in the vacuum have found a small but stable parameter

space for the mixing angle and mass-squared difference [8]: $\sin^2 2\Theta_{21} = 0.59...1.0$ and $\Delta m^2_{21} = (0.47...9.8)\times 10^{-11}$ eV^2, with respect to changes of the total fluxes of ^{8}B and ^{7}Be neutrinos. This implies that our model favors the solution of long-wavelength vacuum oscillations to the solar neutrino problem. In addition, $\Delta m^2_{21} \sim 10^{-11}$ eV^2 implies that the neutrino mass eigenstates ν_1 and ν_2 are essentially degenerate.

The atmospheric neutrinos originate from cosmic ray showers in the upper atmosphere, in particular from pion and subsequent muon decays. Depletion of ν_μ relative to ν_e has been observed in several experiments [9]. In the assumption that this problem is solved by $\nu_\mu - \nu_\tau$ oscillations, the combined data indicate $\Delta m^2_{32} \sim 10^{-2}$ eV^2 and $\sin^2 2\Theta_{32} = 0.4...1.0$. From eq. (13) one obtains

$$P_{\mu\mu} \; = \; 1 - \frac{1}{9}\sin^2 F_{21} - \frac{4}{9}\sin^2 F_{31} - \frac{4}{9}\sin^2 F_{32} \; . \tag{15a}$$

Since we have $L \leq 1.3 \times 10^4$ km and $|\mathbf{P}| \geq 0.2$ GeV, $\Delta m^2_{21} \sim 10^{-11}$ eV^2 obtained above implies that the $\sin^2 F_{21}$ term in eq. (15a) can be safely neglected. Note that the extreme smallness of Δm^2_{21} leads to $\Delta m^2_{32} \approx \Delta m^2_{31}$ (i.e. $F_{32} \approx F_{31}$), and then we obtain:

$$P_{\mu\mu} \; \approx \; 1 - \frac{8}{9}\sin^2 F_{32} \; , \tag{15b}$$

i.e. the deficit of ν_μ mainly arises from $\nu_\mu - \nu_\tau$ oscillations. This result corresponds to the mixing magnitude $\sin^2 2\Theta_{32} \approx 8/9$, which is consistent with the experimental constraints. Accordingly we have $\Delta m^2_{32} \sim 10^{-2}$ eV^2, as indicated by the allowed region on the $\Delta m^2_{32} - \sin^2 \Theta_{32}$ plot [9].

Let N_e and N_μ be the original ν_e and ν_μ fluxes respectively, at the point of production somewhere in the atmosphere. Due to oscillations after travelling a distance, the effective fluxes $\hat{N}_e$ and $\hat{N}_\mu$ at the point of detection are given as

$$\hat{N}_e \; = \; N_e \left(P_{ee} + \frac{N_\mu}{N_e} P_{e\mu} \right) \; = \; N_e \left[1 - \left(1 - \frac{1}{3}\frac{N_\mu}{N_e} \right) \sin^2 F_{21} \right] \tag{16a}$$

and

$$\begin{aligned} \hat{N}_\mu \; &= \; N_\mu \left(P_{\mu\mu} + \frac{N_e}{N_\mu} P_{\mu e} \right) \\ &= \; N_\mu \left[1 - \frac{1}{3}\left(\frac{1}{3} - \frac{N_e}{N_\mu} \right) \sin^2 F_{21} - \frac{4}{9}\sin^2 F_{31} - \frac{4}{9}\sin^2 F_{32} \right] \; . \end{aligned} \tag{16b}$$

Here $N_\mu / N_e \approx 2$, obtained from the Monte Carlo calculations for low-energy ν_μ and ν_e fluxes [10]. To the approximation made in eq. (15b), we obtain $\hat{N}_e \approx N_e$ and

$$\frac{\hat{N}_\mu / N_\mu}{\hat{N}_e / N_e} \; \approx \; 1 - \frac{8}{9}\sin^2 F_{32} \; , \tag{17}$$

which can be confronted with the experimental data [9].

In our approach we find that the neutrino mass eigenstates ν_1 and ν_2 are essentially degenerate. Furthermore, due to the constraint $\Delta m^2_{32} \approx \Delta m^2_{31} \sim 10^{-2}$ eV^2, all three neutrino states

must be nearly degenerate. If one identifies the dark matter of the universe (or at least its hot dark matter component) with neutrino matter, we must have $m_1 + m_2 + m_3 \approx 3m_1 \approx 7...25$ eV. For example, if $m_i \approx 2.5$ eV, one has $m_1 + m_2 + m_3 \approx 7.5$ eV. Recently a related case has been discussed in ref. [11].

Within our approach, the degeneracy of three neutrino masses is suggestive that the mass generation of neutrinos proceeds in three steps. At the first step a diagonal universal mass is introduced: $|\varepsilon_\nu| = |\varrho_\nu| = |\delta_\nu|$. Secondly, the degeneracy between ν_3 and ν_2 is lifted: $|\varepsilon_\nu| \neq |\varrho_\nu| = |\delta_\nu|$. Finally, a tiny difference between $|\varrho_\nu|$ and $|\delta_\nu|$ appears.

We mentioned before that the neutrino mass terms could be either of Fermi-Dirac type or of Majorana type. In our approach the neutrino mass terms correspond to the small breaking terms in the charged lepton sector, i.e. they are expected to be small; and it does not seem necessary for us to consider special mechanisms like the so-called "see-saw" mechanism [12] for Majorana states to suppress the neutrino masses. Thus we see no particular strong reason why the neutrino mass terms should be of Majorana type.

In the case of Majorana masses, the difficulty arises to fulfill the bound $\langle m \rangle \leq 0.7$ eV for neutrinoless $\beta\beta$-decay [13], where $\langle m \rangle$ is an effective mass factor. It is known that the $(\beta\beta)_{0\nu}$-decay amplitude depends on the masses of Majorana neutrinos m_i and on the elements of the lepton mixing matrix V_{ek}:

$$\langle m \rangle \sim \sum_{k=1}^{3} \left[V_{ek}^2 m_k \lambda(\psi_k) \right] , \tag{18}$$

where $\lambda(\psi_k)$ is the CP parity of the Majorana field ψ_k. If $\lambda(\psi_1) = +1$ and $\lambda(\psi_2) = -1$ (or vice versa), one finds that $\langle m \rangle \sim (m_1 - m_2)/2$, which is considerably suppressed due to the nearly degeneracy of m_1 and m_2. This implies that there may exist two Majorana neutrinos with opposite CP eigenvalues, and their relative CP parities are in principle observable in $(\beta\beta)_{0\nu}$-decay [14]. Within our approach this possibility exists. Thus a degenerate Majorana mass of about 2.5 eV for all three neutrinos need not be in conflict with the data on neutrinoless $\beta\beta$-decay.

It is of interest to note that the required cancellation in $\langle m \rangle$ takes place in the specific case $\varrho_\nu \approx -\delta_\nu$, parallel to the condition $\varrho_l = -\delta_l$ taken above. In this sense, one is invited to speculate about a similarity of the perturbative structures of the charged lepton and neutrino mass matrices, although they are obviously very different in the "democratic limit".

In our model for the neutrino masses the mismatch between the "democratic" mass matrix of the charged leptons and the neutrino mass matrix, where the "democratic" mass term is absent, generates large effects of flavor mixing. The associated mixing angles are large, calculable

and in lowest order independent of the lepton masses. The neutrino masses consistent with the experimental constraints are nearly degenerate. One may interpret the hot dark matter component of the cosmic mass density as neutrino matter, in which the three neutrinos are expected to have masses of the order of a few eV. If our description of the neutrino mass and mixing pattern is correct, we expect that a positive signal for neutrino oscillations will be obtained by investigating a ν_μ-beam at distances of the order of several hundred kilometers from its production point [15]. In our view it would be of high interest to carry out such long base-line experiments.

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